Solution **Ans. 1.5**

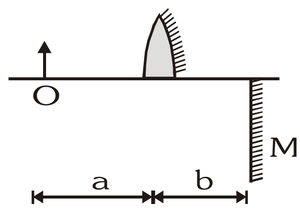
For case (a) $\frac{\mu}{v_1}-\frac{1}{\infty}=\frac{\mu-1}{R}$, $\frac{1}{v_2}-\frac{\mu}{v_1-R}=\frac{1-\mu}{\infty}$ and $v_2=\frac{2R}{m}$ $\Rightarrow m=2(\mu-1)\mu$

For case (b) $\frac{1}{v_2}-\frac{\mu}{\infty}=\frac{1-\mu}{-R}$ and $v_2=\frac{R}{m-1}$ $\Rightarrow$ $\mu = m$

Therefore $\mu=2(\mu-1)\mu \Rightarrow$ $\mu-1=\frac{1}{2}$ $\Rightarrow \mu = 1.5$

Example#27

In figure, L is half part of an equiconvex glass lens (μ = 1.5) whose surfaces have radius of curvature R = 40 cm and its right surface is silvered. Normal to its principal axis a plane mirror M is placed on right of the lens. Distance between lens L and mirror M is b. A small object O is placed on left of the lens such that there is no parallax between final images formed by the lens and mirror. If transverse length of final image formed by lens is twice that of image formed by the mirror, calculate distance 'a' in cm between lens and object.

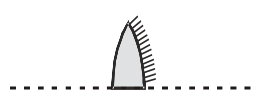


Solution **Ans. 5**

Distance of image of object O from plane mirror = a + b. Since, there is no parallax between the images formed by the silvered lens L and plane mirror M, therefore, two images are formed at the same point. Distance of image = (a + 2b) behind lens. Since, length of image formed by L is twice the length of image formed by the mirror M and length of image formed by a plane mirror is always equal to length of the object, therefore, transverse magnification produced by the lens L is equal to 2.Since, distance of object from L is a, therefore, distance of image from L must be equal to 2a.

$$\therefore (a + 2b) = 2a \Rightarrow b = \frac{a}{2}$$

The silvered lens L may be assumed as a combination of an equi–convex lens and a concave mirror placed in contact with each other co–axially as shown in figure.



Focal length of convex lens f_1 is given by $\frac{1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \Rightarrow f_1 = 40$ cm

For concave mirror focal length, $f_m=\frac{R}{2}=-20$ cm

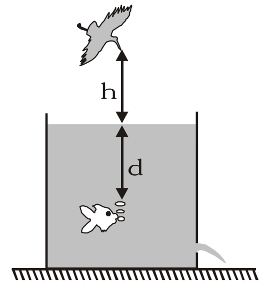
The combination L behaves like a mirror whose equivalent focal length F is given by

$$\frac{1}{F}=\frac{1}{f_m}-\frac{2}{f_1} \Rightarrow F = -10 \text{ cm},$$

Hence, for the combination u = –a, v = +2a, F = –10 cm

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{F} \Rightarrow a = 5$ cm

Distance of Bird as seen by fish $x_{Bf} = d + \mu h$



By differentiating $\frac{d(x_{fB})}{dt} = \frac{dh}{dt} + \frac{1}{\mu}\frac{d(d)}{dt}$

$v_{FB} \Rightarrow 3 + \frac{4}{\mu} = 6$ cm/s $\quad \left[\frac{dh}{dt} = 5 - 2 = 3\,cm/s, \frac{d(d)}{dt} = 2 - (-2) = 4\,cm/s, \mu = \frac{4}{3}\right]$

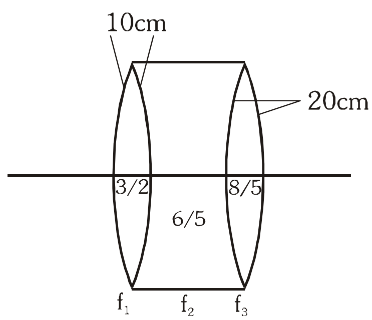
$v_{BF} = \left(\frac{d(d)}{dt}\right) + \mu\left(\frac{dh}{dt}\right) \Rightarrow 4 + \left(\frac{4}{3}\right)(3) \Rightarrow 8$ cm/s

$\frac{d(d)}{dt}$ (for fish image after reflection = 0) $\Rightarrow 3 + \frac{1}{\mu}(0) = 3$ cm/s

Similarly speed of image of bird $\Rightarrow 4$ cm/s

Example#25

In the shown figure the focal length of equivalent system in the form of $\left(\frac{50x}{13}\right)$. Find the value of x.



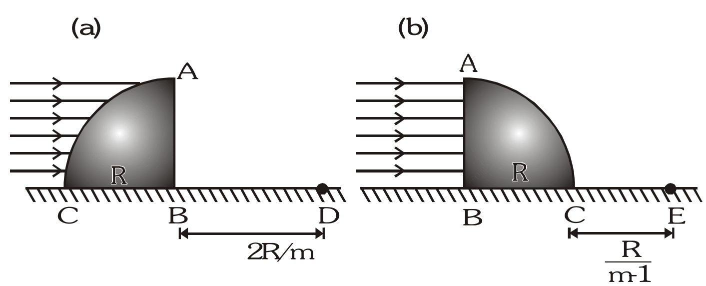
Ans. 2

$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} + \frac{1}{10}\right) = \frac{1}{10}$; $\frac{1}{f_2} = \left(\frac{6}{5} - 1\right)\left(\frac{-1}{10} - \frac{1}{20}\right) = \frac{-3}{100}$ and $\frac{1}{f_3} = \left(\frac{8}{5} - 1\right)\left(\frac{1}{20} + \frac{1}{20}\right) = \frac{3}{50}$

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{10} + \frac{-3}{100} + \frac{3}{50} = \frac{100}{13} = \frac{50x}{13} \Rightarrow$ x=2

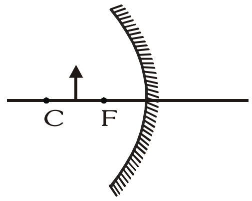
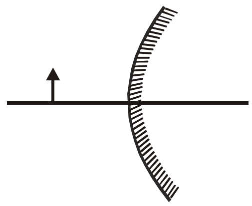
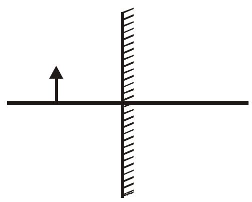
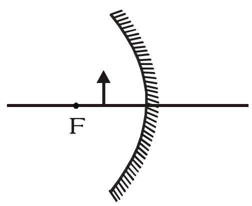
Example#26

Quarter part of a transparent cylinder ABC of radius R is kept on a horizontal floor and a horizontal beam of light falls on the cylinder in the two different arrangement of cylinder as shown in the figure (a) & (b). In arrangement (a) light converges at point D, which is at a distance 2R/m from B. And in arrangement (b) light converges at point E, which is at a distance R/(m – 1) from C. Find out the refractive index of the material.



Example#23

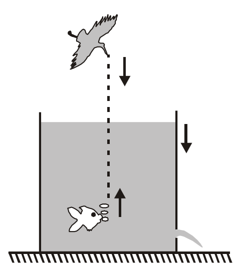
Column-I contains a list of mirrors and position of object. Match this with Column-II describing the nature of image.

Column I		Column II	
(A)		(P)	real, inverted, enlarged
(B)		(Q)	virtual, erect, enlarged
(C)		(R)	virtual, erect, diminished
(D)		(S)	virtual, erect

Solution Ans. (A) P (b)RS (C) S (D) QS

Example#24

A bird in air is diving vertically over a tank with speed 5 cm/s, base of tank is silvered. A fish in the tank is rising upward along the same line with speed 2 cm/s. Water level is falling at rate of 2 cm/s. [Take : $\mu_{water} = 4/3$]



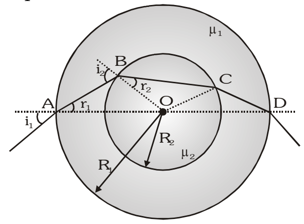
Column I (cm/s)			Column II
(A)	Speed of the image of fish as seen by the bird directly	(P)	8
(B)	Speed of the image of fish formed after reflection in the mirror as seen by the bird	(Q)	6
(C)	Speed of image of bird relative to the fish looking upwards	(R)	3
(D)	Speed of image of bird relative to the fish looking downwards in the mirror	(S)	4

Solution Ans. (A) – (Q) ; (B) – (R) ; (C) – (P) ; (D) – (S)

Distance of fish asseen by bird $x_{fB} = h + \frac{d}{\mu}$

Example#19 to 21

There is a spherical glass ball of refractive index μ_1 and another glass ball of refractive index μ_2 inside it as shown in figure. The radius of the outer ball is R_1 and that of inner ball is R_2. A ray is incident on the outer surface of the ball at an angle i_1.



19. Find the value of r_1

(A) $\sin^{-1}\left(\dfrac{\sin i_1}{\mu_1}\right)$ (B) $\sin^{-1}(\mu_1 \sin i_1)$ (C) $\sin^{-1}\left(\dfrac{\mu_1}{\sin i_1}\right)$ (D) $\sin^{-1}\left(\dfrac{1}{\mu_1 \sin i_1}\right)$

20. Find the value of i_2

(A) $\sin^{-1}\left(\dfrac{R_2}{R_1}\dfrac{\sin i_1}{\mu_1}\right)$ (B)]$\sin^{-1}\left(\dfrac{R_1}{R_2}\dfrac{\sin i_1}{\mu_2}\right)$ (C) $\sin^{-1}\left(\dfrac{R_1}{R_2}\dfrac{\sin i_1}{\mu_1}\right)$ (D) $\sin^{-1}\left(\dfrac{R_2}{R_1}\dfrac{\sin i_1}{\mu_2}\right)$

21. Find the value of r_2

(A) $\sin^{-1}\left(\dfrac{R_1}{\mu_2 R_2}\sin i_1\right)$ (B) $\sin^{-1}\left(\dfrac{R_2}{\mu_2 R_1}\sin i_1\right)$ (C) $\sin^{-1}\left(\dfrac{R_1}{\mu_1 R_2}\dfrac{1}{\sin i_1}\right)$ (D) $\sin^{-1}\left(\dfrac{R_2}{\mu_1 R_1}\sin i_1\right)$

Solution

19. Ans. (A)

$\mu_1 \sin r_1 = \sin i_1 \Rightarrow r_1 = \sin^{-1}\left(\dfrac{\sin i_1}{\mu_1}\right)$

20. Ans. (C)

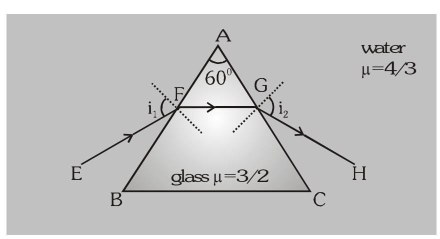
Using sine rule $\dfrac{\sin r_1}{R_2} = \dfrac{\sin(180-i_2)}{R_1} \Rightarrow \sin i_2 = \dfrac{R_1}{R_2}\sin r_1 = \left(\dfrac{R_1}{R_2}\dfrac{\sin i_1}{\mu_1}\right) \Rightarrow i_2 = \sin^{-1}\left(\dfrac{R_1}{R_2}\dfrac{\sin i_1}{\mu_1}\right)$

21. Ans. (A)

$\dfrac{\mu_1}{\mu_2}\sin i_2 = \sin r_2;\ \dfrac{\mu_1}{\mu_2}\dfrac{R_1}{R_2}\dfrac{\sin i_1}{\mu_1} = \sin r_2;\ r_2 = \sin^{-1}\left(\dfrac{R_1}{\mu_2 R_2}\sin i_1\right)$

Example#22

Consider a an equilateral prism ABC of glass $\left(\mu = \dfrac{3}{2}\right)$ placed in water $\left(\mu = \dfrac{4}{3}\right)$



Column-I		Column-II	
(A)	FG is parallel to BC	(P)	Maximum deviation
(B)	$i_1 = 90^0$	(Q)	Minimum deviation
(C)	$i_1 = i_2 = \sin^{-1}\left(\dfrac{9}{16}\right)$	(R)	TIR will take place at surface AC
(D)	EF is perpendicular to AB	(S)	No TIR will take place at surface BC

Solution **Ans. (A) QS, (B) PS, (C) QS, (D) S**

At minimum deviation $i_1 = i_2$, $EF \parallel BC$; At maximum deviation $i_1 = 90^0$ or $i_2 = 90^0$

For $i_1 = 0$, TIR will not take place at AC

Solution

13. Ans. (A)

$$\text{Time } t = \frac{\text{distance}}{\text{speed}} = \frac{\sqrt{Y_1^2 + x^2} + \sqrt{(\ell - x)^2 + Y_2^2}}{c}$$

14. Ans. (A)

For least time $\frac{dt}{dx} = 0 \Rightarrow \frac{2x}{\sqrt{Y_1^2 + x^2}} - \frac{2(\ell - x)}{\sqrt{(\ell - x)^2 + Y_2^2}} = 0 \Rightarrow \sin\theta_1 = \sin\theta_2 \Rightarrow \theta_1 = \theta_2$

15. Ans. (A)

Example#16 to 18

One hard and stormy night you find yourself lost in the forest when you come upon a small hut. Entering it you see a crooked old woman in the corner hunched over a crystal ball. You are about to make a hasty exit when you hear the howl of wolves outside. Taking another look at the gypsy you decide to take your chances with the wolves, but the door is jammed shut. Resigned to a bad situation you approach her slowly, wondering just what is the focal length of that nifty crystal ball.

16. If the crystal ball is 20 cm in diameter with R.I. = 1.5, the gypsy lady is 1.2 m from the ball, where is the image of the gypsy in focus as you walk towards her?

(A) 6.9 cm from the crystal ball (B) 7.9 cm from the crystal ball
(C) 8.9 cm from the crystal ball (D) None of these

17. The image of old lady is

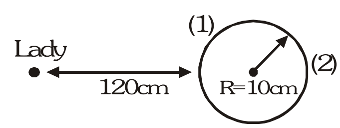
(A) real, inverted and enlarged (B) erect, virtual and small
(C) erect, virtual and magnified (D) real, inverted and small

18. The old lady moves the crystal ball closer to her wrinkled old face. At some point you can no longer get an image of her. At what object distance will there be no change of the gypsy formed?

(A) 10cm (B) 5 cm (C) 15 cm (D) None of these

Solution

16. Ans. (A)



For refraction at 1st surface $\frac{1.5}{v_1} - \frac{1}{-120} = \frac{1.5-1}{+10} \Rightarrow v_1 = 36\text{cm}$

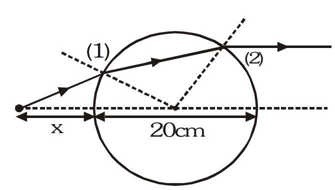
for refraction at 2nd surface $\frac{1}{v} - \frac{1.5}{(36-20)} = \frac{1-1.5}{-10} \Rightarrow v = \frac{80}{11.5} = 6.9\text{cm}$

17. Ans. (D)

Total magnification = $m_1 m_2 = \left(\frac{\mu_1 v_1}{\mu_2 u}\right)\left(\frac{\mu_2 v}{\mu_1 v_1}\right) = \frac{v}{u}$ = negative

18. Ans. (B)

At this point image will formed at infinity

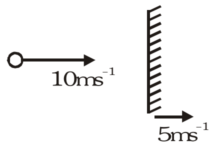


For refraction at second surface $\frac{1}{\infty} - \frac{1.5}{v_1} = \frac{1-1.5}{-10} \Rightarrow v_1 = -30\text{cm}$

For refraction at first surface $\frac{1.5}{-10} - \frac{1}{-x} = \frac{1.5-1}{+10} \Rightarrow x = 5\text{cm}$

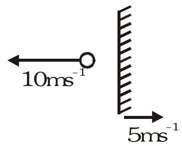
Solution **Ans.(AC)**

Case I



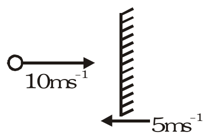
$\vec{v}_{om} = 5\hat{i}, \vec{v}_{Im} = -5\hat{i} \Rightarrow v_I = 0$

Case II



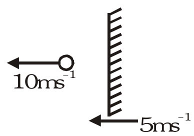
$\vec{v}_{om} = -15\hat{i}, \vec{v}_{Im} = +15\hat{i} \Rightarrow v_I = 20ms^{-1}$

Case II



$\vec{v}_{om} = 15\hat{i}, \vec{v}_{Im} = -15\hat{i} \Rightarrow v_I = 20ms^{-1}$

Case IV

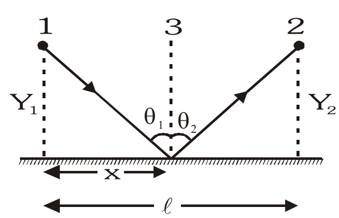


$\vec{v}_{om} = -5\hat{i}, \vec{v}_{Im} = +15\hat{i} \Rightarrow v_I = 0\ ms^{-1}$

OR $\quad v_I = \left|2\vec{v}_m - \vec{v}_o\right| = \left|2(\pm 5) - (\pm 10)\right| = 20$ or 0 m/s

Example#13 to 15

A ray of light travelling with a speed c leaves point 1 shown in figure and is reflected to point 2. The ray strikes the reflecting surface at a distance x from point 1. According to Fermat's principle of least time, among all possible paths between two points, the one actually taken by a ray of light is that for which the time taken is the least (In fact there are some cases in which the time taken by a ray is maximum rather than a minimum).



13. Find the time for the ray to reach from point 1 to point 2.

(A) $\dfrac{\sqrt{Y_1^2 + x^2} + \sqrt{(\ell - x)^2 + Y_2^2}}{c}$ (B) $\dfrac{\ell}{c}$ (C) $\dfrac{Y_1}{c} + \dfrac{Y_2}{c}$ (D) $\dfrac{2x}{c}$

14. Under what condition is time taken least?

(A) $\theta_1 = \theta_2$ (B) $x = \ell - x$ (C) $Y_1 = Y_2$ (D) all of these

15. Which of the following statement is in accordance with Fermat's principle

(A) A ray as it moves from one point to another after reflection takes shortest possible path

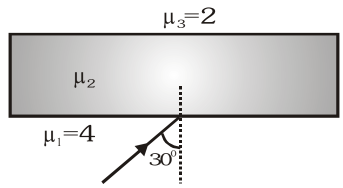
(B) A ray as it moves from one point to another after reflection takes longest possible path

(C) A ray as it moves from one point to another takes shortest possible time

(D) A ray as it moves from one point to another takes longest possible time

Example#10

A ray of light is incident in situation as shown in figure.



Which of the following statements is/are true?

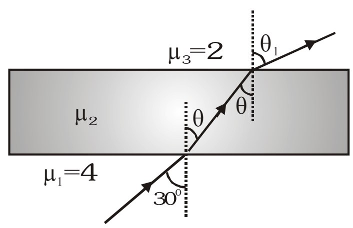
(A) If $\mu_2 = 3.2$ then the angle of deviation is zero

(B) If $\mu_2 = 2.8$ then the angle of deviation is 60^0

(C) If $\mu_2 = 1.8$ then the angle of deviation is 120^0

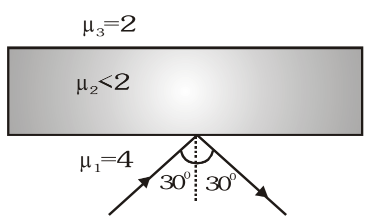
(D) If $\mu_2 = 1.8$ then the angle of deviation is 60^0

Solution **Ans.(BC)**



$\mu_1 \ \sin 30^0 = \mu_2 \sin\theta = \mu_3 \sin\theta_1 \Rightarrow 2 = \mu_2 \sin\theta = 2 \ \sin\theta_1 \Rightarrow \theta_1 = 90^0$ and $\mu_2 > 2$

For μ_2 <2, TIR will take place at first surface.



$\Rightarrow \delta = 120^0$

Example#11

A fish lies at the bottom of a 4m deep water lake. A bird flies 6 m above the water surface and refractive index of water is 4/3. Then the distance between

(A) Bird and image of fish is 9 m

(B) Fish and image of bird is 12 m

(C) Fish and image of bird is 8m

(D) Fish and image of bird is 10m

Solution **Ans. (AB)**

For a bird, fish appears 3 m below the water surface and for fish, bird appears 9m above the surface.

Example#12

A plane mirror and an object has speeds of 5 m/s and 10 m/s respectively. If the motion of mirror and object is along the normal of the mirror then the speed of image may be :–

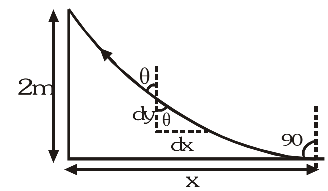
(A) 0 m/s (B) 10 m/s (C) 20 m/s (D) 25 m/s

Example#7

A man of height 2 m stands on a straight road on a hot day. The vertical temperature in the air results in a variation of refractive index with height y as $\mu = \mu_0\sqrt{(1+ay)}$ where μ_0 is the refractive index of air near the road and $a = 2 \times 10^{-6}$/m. What is the actual length of the road, man is able to see

(A) 2000 m (B) 390 m (C) infinite distance (D) None of these

Solution **Ans. (A)**

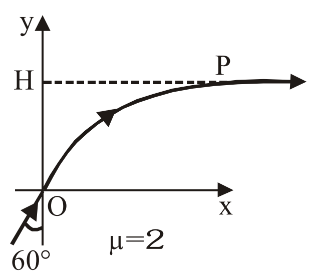


$\mu \sin\theta = \mu_0 \sin 90 = \mu_0 \Rightarrow \sin\theta = \dfrac{\mu_0}{\mu} = \dfrac{1}{\sqrt{1+ay}}$

But $\dfrac{dx}{dy} = \tan\theta \Rightarrow dx = (dy)\left(\dfrac{1}{\sqrt{ay}}\right) \Rightarrow x = \dfrac{1}{\sqrt{a}}\int_0^2 \dfrac{1}{\sqrt{y}}dy = \left[2\sqrt{\dfrac{y}{a}}\right]_0^2 = 2000\text{m}$

Example#8

A system of coordinates is drawn in a medium whose refractive index varies as $\mu = \dfrac{2}{1+y^2}$, where $0 \le y \le 1$ and $\mu = 2$ for $y < 0$ as shown in figure. A ray of light is incident at origin at an angle 60 with y–axis as shown in the figure. At point P ray becomes parallel to x-axis. The value of H is :-



(A) $\left\{\left(\dfrac{2}{\sqrt{3}}\right) - 1\right\}^{1/2}$ (B) $\left\{\dfrac{2}{\sqrt{3}}\right\}^{1/2}$ (C) $\left\{\left(\sqrt{3}\right) - 1\right\}^{1/2}$ (D) $\left(\dfrac{4}{\sqrt{3}} - 1\right)^{1/2}$

Ans. (A)

$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 \Rightarrow$ at origin x = 0, y = 0 $\Rightarrow \mu = 2$ & $\theta = 60$

at point P : $\theta = 90 \Rightarrow 2\sin 60 = \mu \sin 90 \Rightarrow \sqrt{3} = \dfrac{2}{y^2+1} \Rightarrow y = \left[\left(\dfrac{2}{\sqrt{3}}\right) - 1\right]^{1/2}$

Example#9

A ray of light is incident along a vector $\hat{i} + \hat{j} - \hat{k}$ on a plane mirror lying in y–z plane. The unit vector along the reflected ray can be

(A) $\dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (B) $\dfrac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$ (C) $\dfrac{-\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (D) $\dfrac{3\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

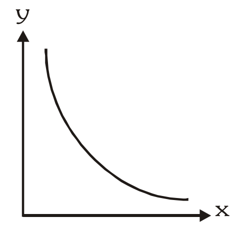
Solution **Ans. (C,D)**

According to law of reflection $\hat{r} = \hat{e} - 2(\hat{e}.\hat{n})\hat{n}$ Here $\hat{e} = \dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$, $\hat{n} = \pm\hat{i}$

so $\hat{e}.\hat{n} = \dfrac{1}{\sqrt{3}} \Rightarrow \hat{r} = \dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}} \pm \dfrac{2\hat{i}}{\sqrt{3}} = \dfrac{3\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ or $\dfrac{-\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

Example#5

If x and y denote the distances of the object and image from the focus of a concave mirror. The line y= 4x cuts the graph at a point whose abscissa is 20 cm. The focal length of the mirror is



(A) 20 cm (B) 40 cm (C) 30 cm (D) can't be determined

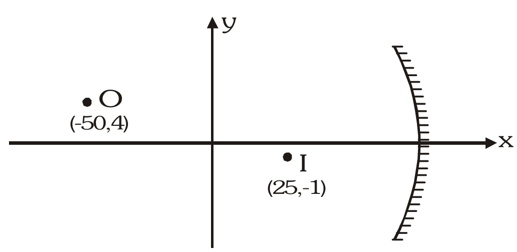
Solution **Ans.(B)**

For x = 20 cm, y = 4 20 = 80 cm

From Newton's formula $xy = f^2 \Rightarrow (2)\ (80) = f^2 \Rightarrow f = 40$ cm

Example#6

A concave mirror forms an image I corresponding to a point object O. The equation of the circle intercepted by the xy plane on the mirror is



(A) $x^2 + y^2 = 1600$

(B) $x^2 + y^2 - 20x - 1600 = 0$

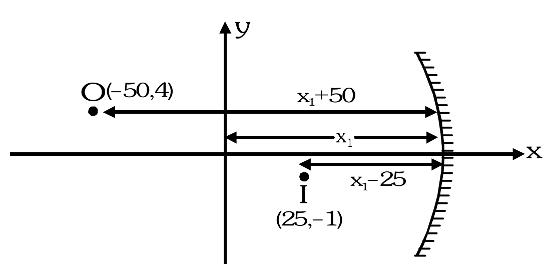
(C) $x^2 + y^2 - 20x - 1500 = 0$

(D) $x^2 + y^2 - 20x + 1500 = 0$

Solution **Ans. (C)**

From mirror equation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{1}{-(x_1 - 25)} + \frac{1}{-(x_1 + 50)} = \frac{1}{f}$ and

from $m = -\frac{v}{u} = -\frac{1}{4}$; $\frac{x_1 - 25}{x_1 + 50} = \frac{1}{4} \Rightarrow 4x_1 - 100 = x_1 + 50 \Rightarrow 3x_1 = 150 \Rightarrow x_1 = 50$ unit



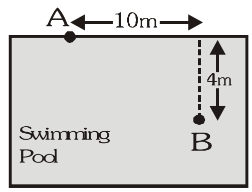
$$\frac{1}{f} = \frac{-1}{50 - 25} + \frac{-1}{50 + 50} = -\left(\frac{1}{25} + \frac{1}{100}\right) \Rightarrow f = -20 \text{ unit} \Rightarrow R = -40 \text{ unit}$$

Centre of circle will be at (10,0)

Equation of required circle $(x-10)^2 + (y-0)^2 = (40)^2 \Rightarrow x^2 + y^2 - 20x - 1500 = 0$

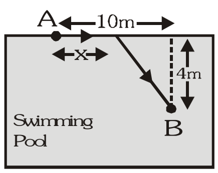
Example#3

On one boundary of a swimming pool, there is a person at point A whose speed of running on ground (boundary) is 10 ms^{-1} , while that of swimming is 6 ms^{-1}. He has to reach a point B in the swimming pool. The distance covered on the boundary so that the time required to reach the point B in the pool is minimum, is–



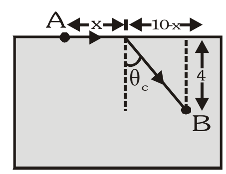
(A) 10m (B) 6m (C) 7m (D) $\sqrt{116}$ m

Solution **Ans. (C)**



Required time $t = \frac{x}{10} + \frac{\sqrt{4^2 + (10-x)^2}}{5}$ For minimum time $\frac{dt}{dx} = 0 \Rightarrow$ x= 7m

OR



Just like light, he has different speeds on ground and in water, so to minimize the time, Fermat's principle must hold good.

$$\sin\theta_C = \frac{6}{10} = \frac{3}{5} \Rightarrow \theta_C = 37^0 \Rightarrow \tan\theta_C = \frac{3}{4} = \frac{10-x}{4} \Rightarrow x = 7m$$

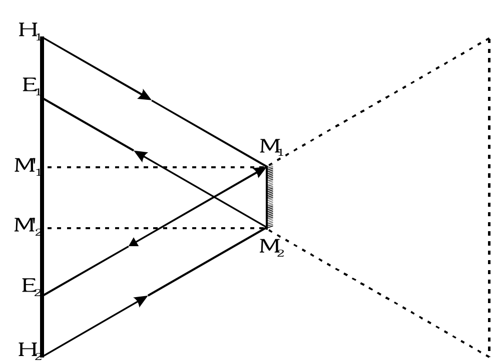
Example#4

A person has D cm wide face and his two eyes are separated by d cm. The minimum width of a mirror required for the person to view his complete face is

(A) $\frac{D+d}{2}$ (B) $\frac{D-d}{4}$ (C) $\frac{D+d}{4}$ (D) $\frac{D-d}{2}$

Solution **Ans. (D)**

According to ray diagram :



$H_1M'_1 = \frac{H_1E_2}{2}$ & $H_2M'_2 = \frac{H_2E_1}{2}$

$H_1E_2 = D - \frac{1}{2}(D-d) = \frac{D+d}{2} = H_2E_1$

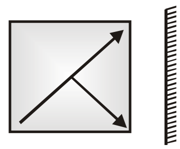
$M'_1M'_2 = D - H_1M'_1 - H_2M'_2 = D - \left(\frac{D+d}{2}\right)$

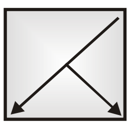
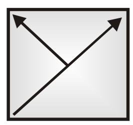
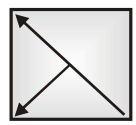
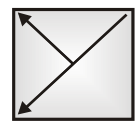
$= \frac{D-d}{2}$

SOME WORKED OUT EXAMPLES

Example#1

The correct mirror–image of the figure given is :-

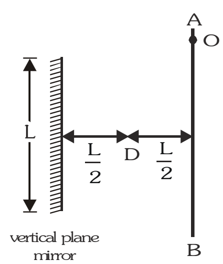


(A)  (B)  (C)  (D) 

Solution **Ans. (C)**

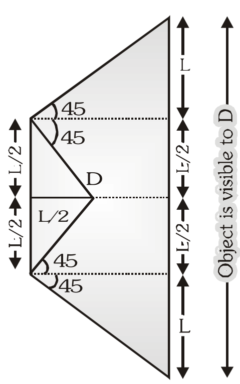
Example#2

A point object O can move along vertical line AB as shown in figure. When image of the object is first visible to D then it is released at t = 0 from rest from A. The time for which image is visible to D is :



(A) $\sqrt{\frac{6L}{g}}$ (B) $\sqrt{\frac{2L}{g}}$ (C) $\sqrt{\frac{3L}{g}}$ (D) $t \to \infty$

Solution **Ans. (A)**



Required time is given by $3L = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{6L}{g}}$

PRESBYOPIA

In this case both near and far object are not clearly visible. To remove this defect two separate spectacles one for myopia and other for hypermetropia are used or bifocal lenses are used.

ASTIGMATISM

In this defect eye can not see object in two orthogonal direction clearly. It can be removed by using cylindrical lens in particular direction.

Example

A person can not see clearly an object kept at a distance beyond of 100 cm. Find the nature and the power of lens to be used for seeing clearly the object at infinity.

Solution

For lens u = $-\infty$ and and v = $-$ 100 cm

$$\therefore \frac{1}{v}-\frac{1}{u}=\frac{1}{f}=\frac{1}{v}=\frac{1}{f} \Rightarrow f=v=-100\text{cm}(\text{concave}) \quad \therefore \text{ Power of lens } P=\frac{1}{f}=-\frac{1}{1}=-1D$$

Example

A far sighted person has a near point of 60 cm. What power lens should be used for eye glasses such that the person can read this book at a distance of 25 cm.

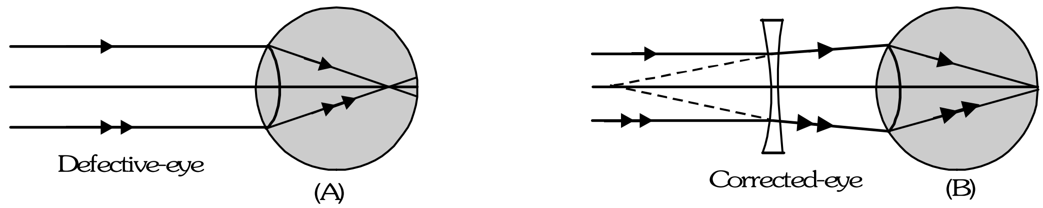
Solution

Here v = $-$ 60 cm, u = $-$25 cm

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=-\frac{1}{60}+\frac{1}{25} \Rightarrow f=\frac{300}{7}\text{cm} \quad \therefore \text{Power}=\frac{1}{f(\text{in m})}=\frac{1}{(3/7)}=+2.33D$$

DEFECTS OF EYES

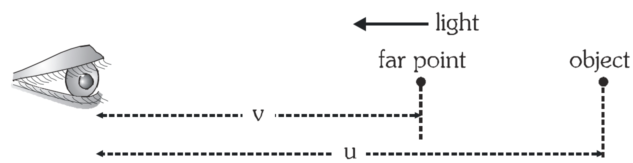
MYOPIA [or Short–sightedness or Near– sightedness]



(i) Distant object are not clearly visible, but near object are clearly visible because image is formed before the retina.

(ii) To remove the defect concave lens is used.

◎ The maximum distance. Which a person can see without help of spectacles is known as far point.



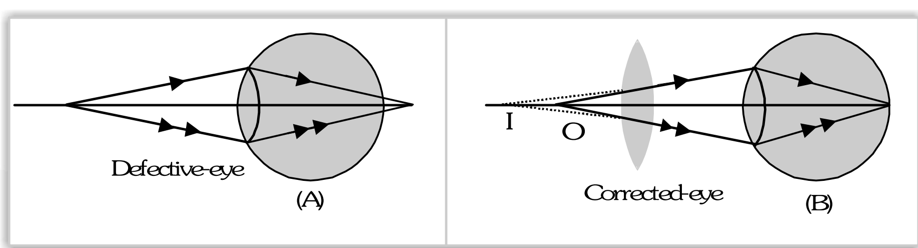
◎ If the reference of object is not given then it is taken as infinity.

◎ In this case image of the object is formed at the far point of person.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=P \Rightarrow \frac{1}{\text{distance of far point (in m)}}-\frac{1}{\text{distance of object (in m)}}=\frac{1}{f}=P$$

$$\frac{100}{\text{distance of far point (in cm)}}-\frac{100}{\text{distance of object (in cm)}}=P$$

LONG–SIGHTEDNESS OR HYPERMETROPIA



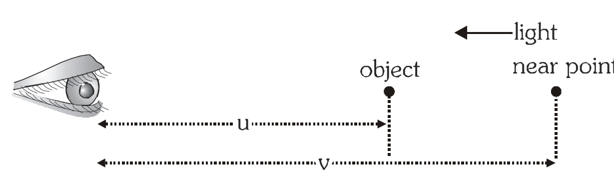
(i) Near object are not clearly visible but far object are clearly visible.

(ii) The image of near object is formed behind the retina.

(iii) To remove this defect convex lens is used.

Near Point :–

The minimum distance which a person can see without help of spectacles.



- In this case image of the object is formed at the near point.
- If reference of object is not given it is taken as 25 cm.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=P \Rightarrow \frac{1}{\text{distance of near point (in m)}}-\frac{1}{\text{distance of object (in m)}}=\frac{1}{f}=P$$

distance of near point = –ve, distance of object = –ve, P = +ve

Example

With diaphragm of the camera lens set at $\frac{f}{2}$, the correct exposure time is $\frac{1}{100}$, then with diaphragm set at $\frac{f}{4}$. Calculate the correct exposure time.

Solution

As exposure time $\propto \frac{1}{(\text{aperture})^2} \Rightarrow t_1 \propto \frac{1}{[f/2]^2}$ and $t_2 \propto \frac{1}{[f/4]^2}$

here $t_1 = \frac{1}{100}$s then $\frac{t_2}{t_1} = \frac{16}{4} = 4 \Rightarrow t_2 = 4t_1 = \frac{4}{100}$s

Example

A good photographic print is obtained by an exposure of two seconds at a distance of 20 cm from the lamp. Calculate the time of exposure required to get an equally good result at a distance of 40 cm.

Solution

We know that the intensity of light varies inversely as the (distance)^2. When distance is doubled, the intensity becomes one–fourth. So, the time of exposure should be four times. Hence, time of exposure = 2 4 = 8 s

Example

Photograph of the ground are taken from an aircraft, flying at an altitude of 2000 m, by a camera with a lens of focal length 50 cm. The size of the film in the camera is 18 cm × 18 cm. What area of the ground can be photographed by this camera at any one time.

Solution

As here u = – 2000m, f = 0.50m, so from lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$,

we have $\frac{1}{v} - \frac{1}{(-2000)} = \frac{1}{0.5} \Rightarrow \frac{1}{v} = \frac{1}{0.5} - \frac{1}{2000} \cong \frac{1}{0.5}\left[\text{as}\ \frac{1}{0.5} >> \frac{1}{2000}\right]$ $\Rightarrow$ v = 0.5m = 50 cm = f

Now as in case of a lens, $m = \frac{v}{u} = \frac{0.5}{-2000} = -\frac{1}{4} \times 10^{-3}$ So I_1 = (ma) (mb) = m^2A [$\because$ A = ab]

$$A = \frac{I_1}{m^2} = \frac{18\text{cm} \times 18\text{cm}}{\left[(1/4) \times 10^{-3}\right]^2} = (720\ \text{m} \times 720\ \text{m})$$

Example

The proper exposure time for a photographic print is 20 s at a distance of 0.6 m from a 40 candle power lamp. How long will you expose the same print at a distance of 1.2 m from a 20 candle power lamp ?

Solution

In case of camera, for proper exposure $I_1D_1^2t_1 = I_2D_2^2t_2$

As here D is constant and I = (L/r^2); $\frac{L_1}{r_1^2} \times t_1 = \frac{L_2}{r_2^2} \times t_2$ So $\frac{40}{(0.6)^2} \times 20 = \frac{20}{(1.2)^2} t \Rightarrow t = 160$ s

Example

A telescope consisting of an objective of focal length 60 cm and a single–lens eyepiece of focal length 5 cm is focussed at a distant object in such a way that parallel rays emerge from the eye piece. If the object subtends an angle of 2 at the objective, then find the angular width of the image.

Solution

$$MP = \frac{f_0}{f_e} = \frac{\beta}{\alpha} \Rightarrow \beta = \alpha\frac{f_0}{f_e} = 2° \times \frac{60}{5} = 24°$$

Example

The focal lengths of the objective and the eye piece of an astronomical telescope are 60 cm and 5 cm respectively. Calculate the magnifying power and the length of the telescope when the final image is formed at (i) infinity, (ii) least distance of distinct vision (25 cm)

Solution

(i) When the final image is at infinity, then :

$$MP = -\frac{f_0}{f_e} = -\frac{60}{5} = -12$$ and length of the telescope is $L = f_0 + f_e = 60 + 5 = 65$ cm

(ii) For least distance of distinct vision, the magnifying power is :

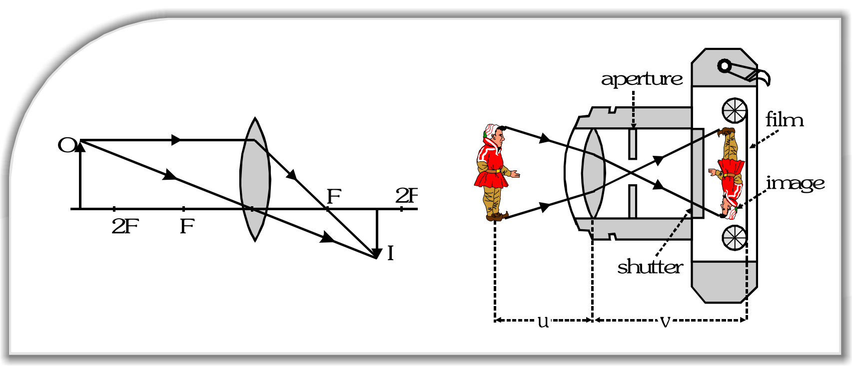
$$MP = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right) = -\frac{60}{5}\left(1 + \frac{5}{25}\right) = -\frac{12 \times 6}{5} = -14.4$$

Now $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e} \Rightarrow \frac{1}{5} = -\frac{1}{25}\frac{1}{u_e} \Rightarrow \frac{-1}{u_e} = \frac{1}{25} + \frac{1}{5} \Rightarrow u_e = -4.17$ cm $\Rightarrow |u_e| = 4.17$ cm

The length of telescope in this position is $L = f_0 + |u_e| = 60 + 4.17 = 64.17$ cm

LENS – CAMERA

There is a convex lens whose aperture and distance from the film can be adjusted. Object is real and placed between ∞ and 2F, so the image is real, inverted diminished and between F and 2F.



If I is the intensity of light, S is the light transmitting area of lens and t is the exposure time, then for proper exposure, I S t = constant

light transmitting area of a lens is proportional to the square of its aperture D ; I D^2 t = constant

If aperture is kept fixed, for proper exposure, I t = constant, i.e., $I_1 t_1 = I_2 t_2$

If intensity is kept fixed, for proper exposure, D^2 t = constant

$$\text{Time of exposure} \propto \frac{1}{(\text{aperture})^2} \quad \dots \text{(i)}$$

The ratio of focal length to the aperture of lens is called f–number of the camera,

$$\text{f–number} = \frac{\text{focal length}}{\text{aperture}} \Rightarrow \text{Aperture} \propto \frac{1}{\text{f – number}} \quad \dots \text{(ii)}$$

From equation (i) and (ii) $\Rightarrow$ Time of exposure $\propto$ (f–number)^2

$$MP = \frac{\text{visual angle with instrument } (\beta)}{\text{visual angle for unaided eye } (\alpha)} \Rightarrow MP = \frac{\frac{h'}{f_0}}{\frac{h'}{-u_e}} = -\frac{f_0}{u_e}[A'B' = h']$$

(i) If the final image is at infinity $v_e = -\infty$, $u_e = -ve$

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow u_e = f_e. \text{ So } MP = -\frac{f_0}{f_e} \text{ and length of the tube } L = f_0 + f_e$$

(ii) If the final image is at D : $v_e = -D$ $u_e = -ve$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right] \text{ So } MP = -\frac{f_0}{u_e} = -\frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right]$$

Length of the tube is $L = f_0 + u_e$

S. No.	Compound – Microscope	S. No.	Astronomical – Telescope
1.	It is used to increase visual angle of near tiny object.	1.	It is used to increase visual angle of distant large objects.
2.	In it field and eye lens both are convergent, of short focal length and aperture.	2.	In it objective lens is of large focal length and aperture while eye lens of short focal length and aperture and both are convergent.
3.	Final image is inverted, virtual and enlarged and at a distance D to ∞ from the eye.	3.	Final image is inverted, virtual and enlarged at a distance D to ∞ from the eye.
4.	MP does not change appreciably if objective and eye lens are interchanged as [MP ~ $(LD / f_0 f_e)$]	4.	MP becomes $(1/m^2)$ times of its initial value if objective and eye–lenses are interchanged as MP ~ $[f_0 / f_e]$
5.	MP is increased by decreasing the focal length of both the lenses.	5.	MP is increased by increasing the focal length of objective lens and by decreasing the focal length of eyepiece
6.	RP is increased by decreasing the wavelength of light used. $\left(\because RP = \frac{2\mu\sin\theta}{\lambda}\right)$	6.	RP is increased by increasing the aperture of objective. $\left(\because RP = \frac{D}{1.22\lambda}\right)$

Example

A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope ? What is the separation between the objective and the eyepiece? When final image is formed at infinity.

Solution

Here, f_0 = 144 cm; f_e = 6.0 cm, MP = ?, L = ?

$$MP = \frac{-f_0}{f_e} = \frac{-144}{6.0} = -24 \text{ and } L = f_0 + f_e = 144 + 6.0 = 150.0 \text{ cm}$$

Example

Diameter of the moon is 3.5×10^3 km and its distance from earth is 3.8×10^5 km. It is seen by a telescope whose objective and eyepiece have focal lengths 4m and 10cm respectively. What will the angular diameter of the image of the moon.

Solution

$$MP = -\frac{f_0}{f_e} = -\frac{400}{10} = -40$$. Angle subtended by the moon at the objective $= \frac{3.5 \times 10^3}{3.8 \times 10^5} = 0.009$ radian.

Thus angular diameter of the image = MP × visual angle = 40 × 0.009 = 0.36 radian = $\frac{0.36 \times 180}{3.14} \simeq 21°$

(ii) When final image is formed at infinity $\frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e}\Rightarrow\frac{1}{\infty}+\frac{1}{u_e}=\frac{1}{f_e}\Rightarrow u_e=f_e$

$$MP=\frac{v_0}{u_0}\left[\frac{D}{f_e}\right]=\frac{f_0}{f_0+u_0}\left[\frac{D}{f_e}\right]=\frac{f_0-v_0}{f_0}\left[\frac{D}{f_e}\right]=\frac{h_2}{h_1}\left[\frac{D}{f_e}\right]$$. Length of the tube $L = v_0 + f_e$

Sign convention for solving numerical u_0 = –ve, v_0 = +ve, f_0 = +ve,
u_e = –ve, v_e = –ve, f_e = +ve, m_0 = –ve, m_e = +ve, M = –ve

Example

A thin convex lens of focal length 5 cm is used as a simple microscope by a person with normal near point (25 cm). What is the magnifying power of the microscope ?

Solution

Here, f = 5 cm; D = 25 cm, M = ? $MP=1+\frac{D}{f}=1+\frac{25}{5}=6$

Example

A compound microscope consists of an objective lens of focal length 2.0 cm and an eye piece of focal length 6.25 cm, separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm) (b) infinity ?

Solution

Here, f_0 = 2.0 cm; f_e = 6.25 cm, u_0 = ?

(a) v_e = – 25 cm $\because \frac{1}{v_e}-\frac{1}{u_e}=\frac{1}{f_e}\ \therefore \frac{1}{u_e}=\frac{1}{v_e}-\frac{1}{f_e}=\frac{1}{-25}-\frac{1}{6.25}=\frac{-1-4}{25}=\frac{-5}{25}\Rightarrow u_e=-5$ cm

As distance between objective and eye piece = 15 cm; v_0 = 15 – 5 = 10 cm

$$\because \frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}\therefore\frac{1}{u_0}=\frac{1}{v_0}-\frac{1}{f_0}=\frac{1}{10}-\frac{1}{2}=\frac{1-5}{10}\Rightarrow u_0=\frac{-10}{4}=-2.5\text{ cm}$$

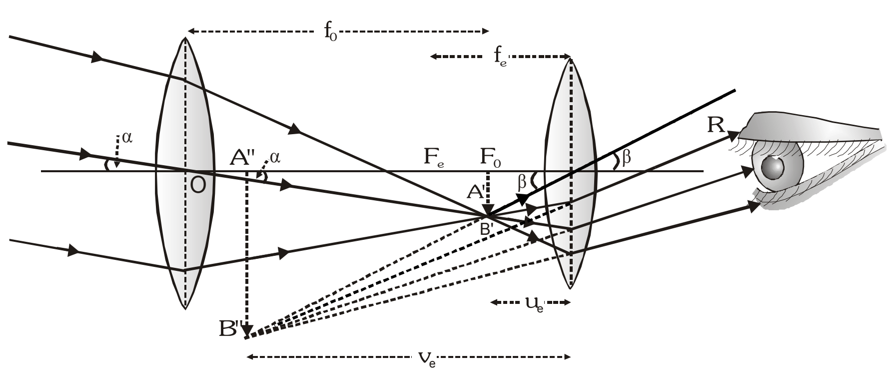
Magnifying power $=\frac{v_0}{|u_0|}\left[1+\frac{D}{f_e}\right]=\frac{10}{2.5}\left[1+\frac{25}{6.25}\right]=20$

(b) $\because v_e=\infty,\ u_e=f_e=6.25$ cm $\quad\therefore v_0 = 15 - 6.25 = 8.75$ cm.

$$\because \frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}\Rightarrow\frac{1}{u_0}=\frac{1}{v_0}-\frac{1}{f_0}=\frac{1}{8.75}-\frac{1}{20}=\frac{2-8.75}{17.5}\Rightarrow u_0=\frac{-17.5}{6.75}=-2.59\text{cm}$$

Magnifying power $=\frac{v_0}{|u_0|}\times\left[1+\frac{D}{f_e}\right]=\frac{v_0}{|u_0|}\times\frac{D}{|u_e|}=\frac{8.75}{2.59}\times\frac{25}{6.25}=13.51$

ASTRONOMICAL TELESCOPE



A telescope is used to see distant object, objective lens forms the image A'B' at its focus. This image A'B' acts as a object for eyepiece and it forms final image A"B".

Example

A man with normal near point 25 cm reads a book with small print using a magnifying glass, a thin convex lens of focal length 5 cm.

(a) What is the closest and farthest distance at which he can read the book when viewing through the magnifying glass ?

(b) What is the maximum and minimum MP possible using the above simple microscope ?

Solution

(a) As for normal eye far and near point are ∞ and 25 cm respectively, so for magnifier $v_{max} = -\infty$ and $v_{min} = -25$ cm. However, for a lens as $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow u = \frac{f}{(f/v) - 1}$

So u will be minimum when v = minimum = –25 cm i.e. $(u)_{min} = \frac{5}{-(5/25) - 1} = -\frac{25}{6} = -4.17\text{cm}$

Ans u will be maximum when v = maximum = ∞ i.e., $u_{max} = \frac{5}{\left(\frac{5}{\infty} - 1\right)} = -5$ cm

So the closest and farthest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

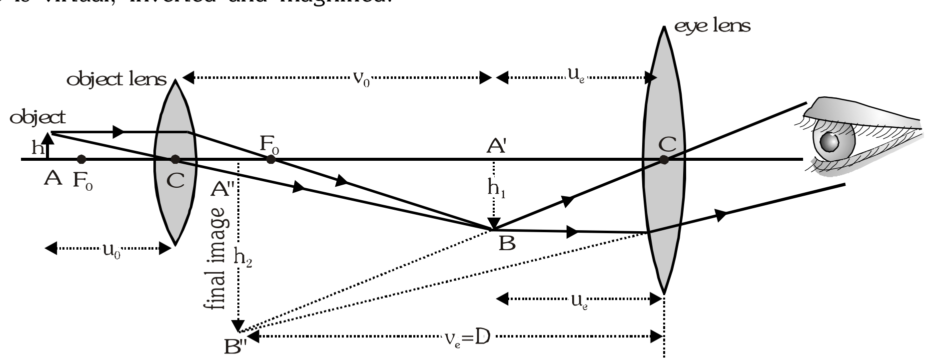
(b) As in case of simple magnifier MP = (D/u). So MP will be minimum when u = max = 5 cm

$\Rightarrow (MP)_{min} = \frac{-25}{-5} = 5\left[= \frac{D}{f}\right]$ and MP will be maximum when u = min = (25/6) cm

$$\Rightarrow (MP)_{max} = \frac{-25}{-(25/6)} = 6\left[= 1 + \frac{D}{f}\right]$$

COMPOUND MICROSCOPE

Compound microscope is used to get more magnified image. Object is placed infront of objective lens and image is seen through eye piece. The aperture of objective lens is less as compare to eye piece because object is very near so collection of more light is not required. Generally object is placed between F – 2F due to this a real inverted and magnified image is formed between 2F – ∞. It is known as intermediate image A'B'. The intermediate image act as a object for eye piece. Now the distance between both the lens are adjusted in such a way that intermediate image falls between the optical centre of eye piece and its focus. In this condition, the final image is virtual, inverted and magnified.



Total magnifying power = Linear magnification angular magnification MP = $m_0 m_e = \frac{v_0}{u_0}\frac{D}{u_e}$

(i) When final image is formed at minimum, distance of distinct vision.

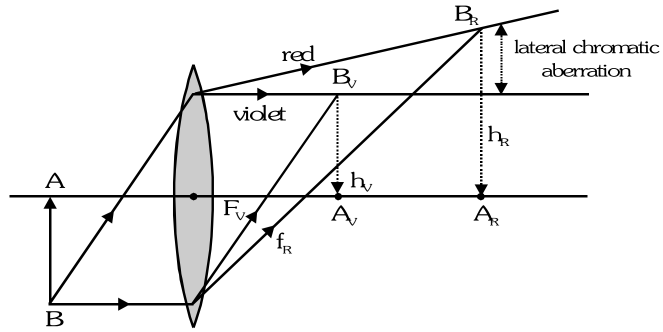
$$MP = \frac{v_0}{u_0}\left[1 + \frac{D}{f_e}\right] = \frac{f_0}{f_0 + u_0}\left[1 + \frac{D}{f_e}\right] = \frac{f_0 - v_0}{f_0}\left[1 + \frac{D}{f_e}\right] = \frac{h_2}{h_1}\left(1 + \frac{D}{f_e}\right)$$

Length of the tube = $v_0 + |u_e|$

LATERAL CHROMATIC ABERRATION

As the focal–length of the lens varies from color to color, the magnification $m = \left[\frac{f}{u+f}\right]$ produced by the lens also varies from color to color.

Therefore, for a finite–size white object AB, the images of different colors formed by the lens are of different sizes.



The formation of images of different colors in different sizes is called lateral chromatic aberration. The difference in the height of the red image $B_R A_R$ and the violet image $B_V A_V$ is known as lateral chromatic aberration. LCA = $h_R - h_V$

ACHROMATISM

If two or more lens combined together in such a way that this combination produce image at a same point then this combination is known as achromatic combination of lenses.

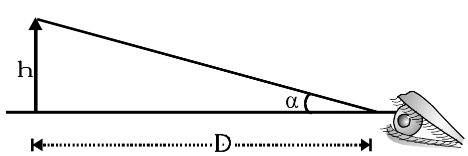
$$\frac{\omega}{f_y}+\frac{\omega'}{f'_y}=0 \Rightarrow \frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0 \Rightarrow \frac{\omega_1}{\omega_2}=-\frac{f_1}{f_2}$$

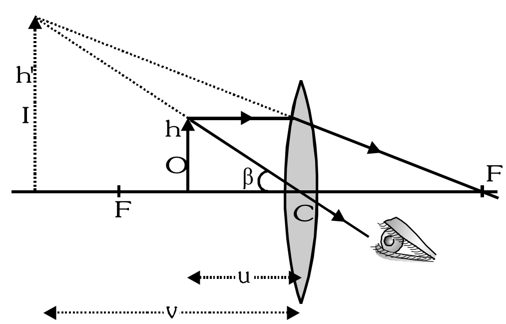
For combination of lens. $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$ (Apply sign convention in numerical)

OPTICAL INSTRUMENTS

Simple microscope

When object is placed between focus and optical centre a virtual, magnified and erect image is formed





$$\text{Magnifying power (MP)} = \frac{\text{visual angle with instrument }(\beta)}{\text{maximum visual angle for unaided eye }(\alpha)} \Rightarrow MP = \frac{\frac{h}{-u}}{\frac{h}{-D}} = \frac{D}{u}$$

(i) When the image is formed at infinity :

by lens equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{-\infty}-\frac{1}{-u}=\frac{1}{f} \Rightarrow u=f$ So $MP=\frac{D}{u}=\frac{D}{f}$

(ii) If the image is at minimum distance of clear vision D :

$\frac{1}{-D}-\frac{1}{-u}=\frac{1}{f} \Rightarrow \frac{1}{u}=\frac{1}{D}+\frac{1}{f}$ [v = –D and u = –ve]

Multiplying by D both the sides $\frac{D}{u}=1+\frac{D}{f} \Rightarrow MP=\frac{D}{u}=1+\frac{D}{f}$

Example

The refractive indices of flint glass for red and violet colours are 1.644 and 1.664. Calculate its dispersive power.

Solution

Here, μ_r = 1.644, μ_v = 1.664, ω = ?

Now $\mu_y = \frac{\mu_v + \mu_r}{2} = \frac{1.664 + 1.644}{2} = 1.654 \quad \because \omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.664 - 1.644}{1.654 - 1} = 0.0305$

Example

In a certain spectrum produced by a glass prism of dispersive power 0.031, it was found that μ_r = 1.645 and μ_v = 1.665. What is the refractive index for yellow colour ?

Solution

Here, ω = 0.031, μ_r = 1.645 μ_v = 1.665, μ_y = ?

$\because \omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \quad \therefore \mu_y - 1 = \frac{\mu_v - \mu_r}{\omega} = \frac{1.665 - 1.645}{0.031} = \frac{0.020}{0.31} = 0.645 \quad \therefore \mu_y = 0.645 + 1 = 1.645$

Example

A combination of two prisms, one of flint and other of crown glass produces dispersion without deviation. The angle of flint glass prism is 15 . Calculate the angle of crown glass prism and angular dispersion of red and violet. (μ for crown glass = 1.52, μ for flint glass = 1.65, ω for crown glass 0.20, ω for flint glass = 0.03).

Solution

Here, A = 15 , A' = ?, ω = 0.03, ω' = 0.02, μ = 1.65, μ' = 1.52, For no deviation, $\delta + \delta' = 0$

$(\mu - 1)A + (\mu' - 1)A' = 0 \Rightarrow (1.65 - 1)15 + (1.52 - 1)A' = 0 \Rightarrow A' = \frac{-0.65 \times 15}{0.52} = -18.75$

Negative sign indicates that two prisms must be joined in opposition. Net angular dispersion

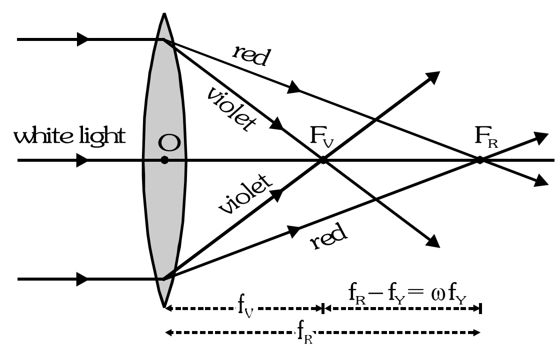
$(\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A' = \omega(\mu - 1)A + \omega'(\mu' - 1)A' = 0.03\,(1.65 - 1)15 + 0.02\,(1.52 - 1)\,(-18.75)$

$= 0.2925 - 0.195 = 0.0975$

CHROMATIC ABERRATION

The image of a object in white light formed by a lens is usually colored and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colors. For a single lens

$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ and as μ of lens is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it. It is defect of lens.



Longitudinal or Axial Chromatic Aberration

When a white object O is situated on the axis of a lens, then images of different colors are formed at different points along the axis. The formation of images of different colors at different positions is called 'axial' or longitudinal chromatic aberration. The axial distance between the red and the violet images $I_R - I_V$ is known as longitudinal aberration. When white light is incident on lens, image is obtained at different point on the axis because focal length of lens depend on wavelength. $f \propto \lambda \Rightarrow f_R > f_V$

$f_R - f_V = \omega f_y \Rightarrow$ **Axial or longitudinal chromatic aberration**

If the object is at infinity, then the longitudinal chromatic aberration is equal to the difference in focal–lengths ($f_R f_V$) for the red and the violet rays.

DISPERSIVE POWER (ω)

It is ratio of angular dispersion (θ) to mean colour deviation (δ_y)

Dispersive power $\omega = \dfrac{\theta}{\delta_y} \Rightarrow \omega = \dfrac{(\mu_V - \mu_R)A}{(\mu_y - 1)A} = \dfrac{\mu_V - \mu_R}{\mu_y - 1} \Rightarrow \omega = \dfrac{\mu_V - \mu_R}{\mu_y - 1}$

Refractive index of mean colour $\mu_y = \dfrac{\mu_V + \mu_R}{2}$. Dispersive power depends only on the material of the prism.

COMBINATION OF PRISM

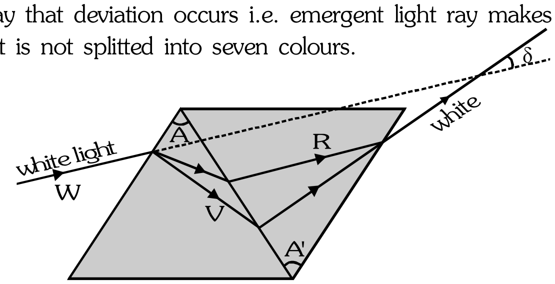
Deviation without dispersion (θ = 0)

Two or more than two thin prism are combined in such a way that deviation occurs i.e. emergent light ray makes angle with incident light ray but dispersion does not occur i.e., light is not splitted into seven colours.

Total dispersion = $\theta = \theta_1 + \theta_2 = (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$

For no dispersion $\theta = 0$; $(\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A' = 0$

Therefore, $A' = -\dfrac{(\mu_V - \mu_R)A}{\mu'_V - \mu'_R}$



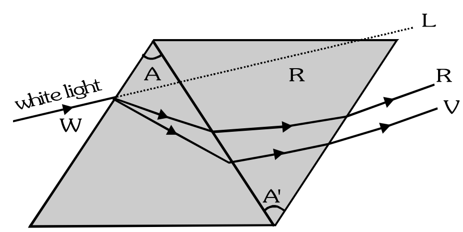
–ve sign indicates that prism angles are in opposite direction.

Dispersion without deviation (δ = 0°)

Two or more than two prisms combine in such a way that dispersion occurs i.e., light is splitted into seven colours but deviation do not occur i.e., emergent light ray becomes parallel to incident light ray.

Total deviation $\delta = \delta_1 + \delta_2$

$\Rightarrow \delta = 0; (\mu - 1)A + (\mu' - 1)A' = 0 \Rightarrow A' = -\dfrac{(\mu - 1)A}{\mu' - 1}$



–ve sign indicates that prism angles are in opposite direction.

GOLDEN KEY POINTS

- Dispersive power like refractive index has no units and dimensions and depends on the material of the prism and is always positive.
- As for a given prism dispersive power is constant, i.e., dispersion of different wavelengths will be different and will be maximum for violet and minimum for red (as deviation is maximum for violet and minimum for red).
- As for a given prism $\theta \propto \delta$ *a* single prism produces both deviation and dispersion simultaneously, i.e., a single prism cannot give deviation without dispersion or dispersion without deviation.

Example

White light is passed through a prism of angle 5 . If the refractive indices for red and blue colours are 1.641 and 1.659 respectively, calculate the angle of dispersion between them.

Solution

As for small angle of prism $\delta = (\mu - 1)A$,

δ_B = (1.659 – 1)   5 = 3.295 and δ_R = (1.641 – 1)   5 =3.205

so $\theta = \delta_B - \delta_R$ = 3.295 – 3.205 = 0.090

Example

Prism angle of a prism is 10°. Their refractive index for red and violet color is 1.51 and 1.52 respectively. Then find the dispersive power.

Solution

Dispersive power of prism $\omega = \left(\dfrac{\mu_v - \mu_r}{\mu_y - 1}\right)$ but $\mu_y = \dfrac{\mu_v + \mu_r}{2} = \dfrac{1.52 + 1.51}{2} = 1.515$

Therefore $\omega = \dfrac{1.52 - 1.51}{1.515 - 1} = \dfrac{0.01}{1.515} = 0.019$

Example

A ray of light passes through an equilateral prism such that angle of incidence is equal of emergence and the later is equal to 3/4th of the angle of prism. Calculate the angle of deviation. Refractive index of prism is 1.5.

Solution

$A = 60$, $\mu = 1.5$; $i_1 = i_2 = \frac{3}{4}A = 45$, $\delta = ?$

$\because A + \delta = i_1 + i_2$ $\quad\therefore 60 + \delta = 45 + 45 \Rightarrow \delta = 90 - 60 = 30$

Example

A prism of refractive index 1.53 is placed in water of refractive index 1.33. If the angle of prism is 60 , calculate the angle of minimum deviation in water. (sin 35.1 = 0.575)

Solution

Here, ${}^a\mu_g = 1.33$, ${}^a\mu_w = 1.53$, $A = 60$, $\delta_m = ?$ ${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1.53}{1.33} = 1.15 \because {}^w\mu_g = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$

$\therefore \frac{\sin(A+\delta_m)}{2} = {}^w\mu_g \times \sin\frac{A}{2} = 1.15\sin\frac{60^\circ}{2} = 0.575 \Rightarrow \frac{A+\delta_m}{2} = \sin^{-1}(0.575) = 35.1$

$\therefore \delta_m = 35.1 \quad 2 - 60 = 10.2$

GOLDEN KEY POINTS

- Angle of prism or refracting angle of prism means the angle between the faces on which light is incident and from which it emerges.
- If the faces of a prism on which light is incident and from which it emerges are parallel then the angle of prism will be zero and as incident ray will emerge parallel to itself, deviation will also be zero, i.e., the prism will act as a transparent plate.
- If μ of the material of the prism is equal to that of surroundings, no refraction at its faces will take place and light will pass through it undeviated, i.e., $\delta = 0$.

DISPERSION OF LIGHT

When white light is incident on a prism then it is splitted into seven colours. This phenomenon is known as dispersion. Prism introduces different refractive index with different wavelength

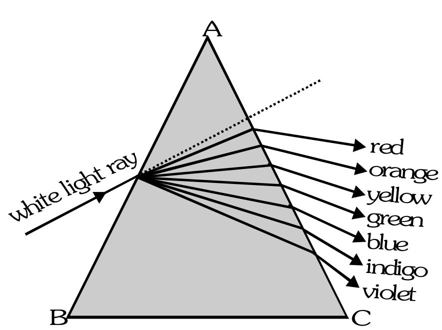
As $\delta_{min} = (\mu - 1)A \because \lambda_R > \lambda_V$ $\quad$ So $\mu_V > \mu_R \Rightarrow \delta_{m(violet)} > \delta_{m(red)}$

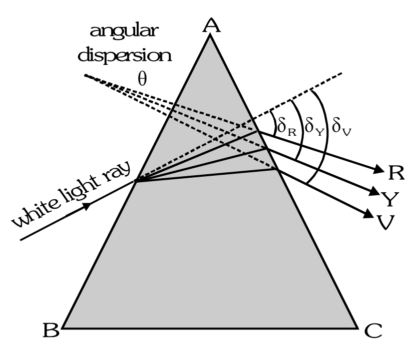
ANGULAR DISPERSION

It is the difference of angle of deviation for violet colour and red colour

Angular dispersion $\theta = \delta_V - \delta_R = (\mu_V - 1)A - (\mu_R - 1)A = (\mu_V - \mu_R)A$

It depends on prism material and on the angle of prism $\theta = (\mu_V - \mu_R)A$





CONDITION OF MINIMUM DEVIATION

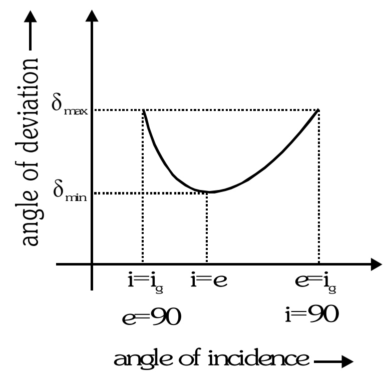
For minimum deviation

In this condition $i_1 = i_2 = i \Rightarrow r_1 = r_2 = r$ and since $r_1 + r_2 = A \;\therefore\; r + r = A \Rightarrow 2r = A \Rightarrow r = \frac{A}{2}$

Minimum deviation $\delta_{min} = 2i - A$; $i = \frac{A+\delta_{min}}{2}$, $r = \frac{A}{2}$

if prism is placed in air $\mu_1 = 1$; 1 sin i = μ sin r

$$\sin\left[\frac{A+\delta_{min}}{2}\right] = \mu \sin\frac{A}{2} \Rightarrow \mu = \frac{\sin\left[\frac{A+\delta_{min}}{2}\right]}{\sin\frac{A}{2}}$$



if angle of prism is small A < 10 then $\sin\theta \approx \theta$

$$\mu = \frac{\frac{A+\delta_{min}}{2}}{\frac{A}{2}} = \frac{A+\delta_{min}}{A} \Rightarrow A + \delta_{min} = \mu A \Rightarrow \delta_{min} = (\mu - 1)A$$

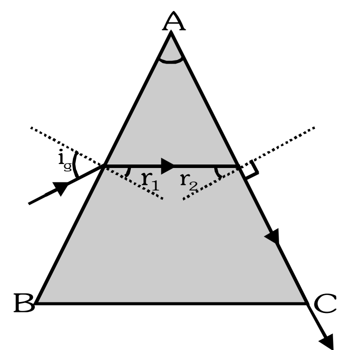
CONDITION FOR MAXIMUM DEVIATION/GRAZING EMERGENCE

◎ **Angle of incidence (i_g)for grazing emergence**

For i_g, e = 90

Applying Snell's law at face AC

$$\mu \sin r_2 = 1 \quad 1 \Rightarrow \sin r_2 = \frac{1}{\mu};\; r_2 = \sin^{-1}\left(\frac{1}{\mu}\right) = \theta_c$$



$r_1 + r_2 = A \Rightarrow r_1 = A - \theta_c$

Again, Applying Snell's law at face AB

$1 \sin i_g = \mu \sin r_1$; $1 \sin i_g = \mu \sin(A - \theta_c)$

$\sin i_g = \mu[\sin A \cos\theta_c - \cos A \sin\theta_c]$

$$i_g = \sin^{-1}\left[\sqrt{\mu^2-1}\,\sin A - \cos A\right] \quad \left[\left(\text{as } \sin\theta_c = \frac{1}{\mu}, \cos\theta_c = \frac{\sqrt{\mu^2-1}}{\mu}\right)\right]$$

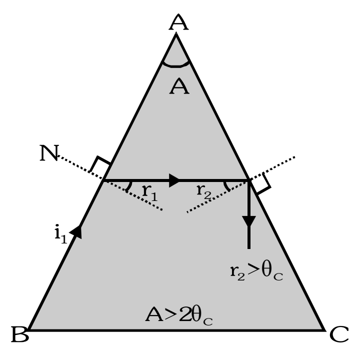
If i increases beyond i_g, r_1 increases thus r_2 decreases and becomes less than θ_c and ray emerges.

Thus $i \geq i_g \Rightarrow$ ray emerges, otherwise TIR. $\delta_{max} = i_g + 90 - A$

NO EMERGENCE CONDITION

Let maximum incident angle on the face AB $i_{max} = 90$

$1 \sin 90 = \mu \sin r_1$; $\sin r_1 = \frac{1}{\mu} = \sin\theta_C$; $r_1 = \theta_C$ ...(i)



if TIR occur at face AC then $r_2 > \theta_C$ ...(ii)

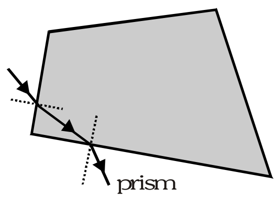
$r_1 + r_2 = A$ ...(iii)

from (i) and (ii) $r_1 + r_2 > \theta_C + \theta_C \Rightarrow r_1 + r_2 > 2\theta_C$ ...(iv)

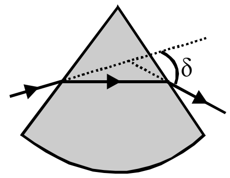
from (iii) and (iv) $A > 2\theta_C \Rightarrow \frac{A}{2} > \theta_C \Rightarrow \sin\frac{A}{2} > \sin\theta_C \Rightarrow \sin\frac{A}{2} > \frac{1}{\mu} \Rightarrow \frac{1}{\sin\frac{A}{2}} < \mu$

PRISM

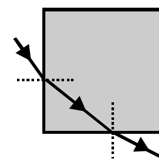
A prism is a homogeneous, transparent medium (such as glass) enclosed by two plane surfaces inclined at an angle. These surfaces are called the 'refracting surfaces' and the angle between them is called the 'refracting angle' or the 'angle of prism'. The section cut by a plane perpendicular to the refracting surfaces is called the 'principal section' of the prism.



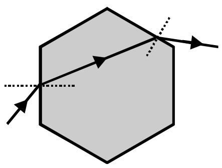
prism



prism

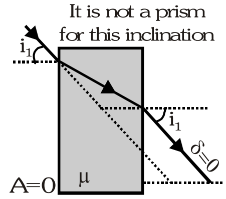


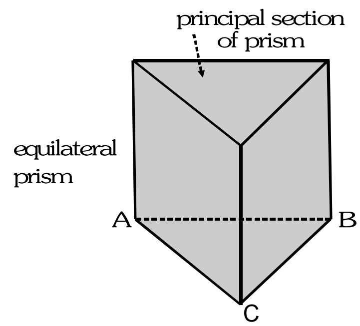
prism

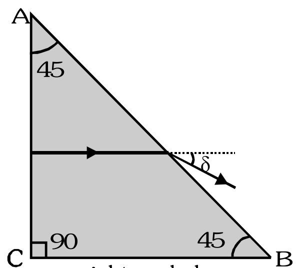


prism

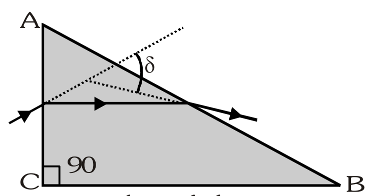
It is not a prism for this inclination





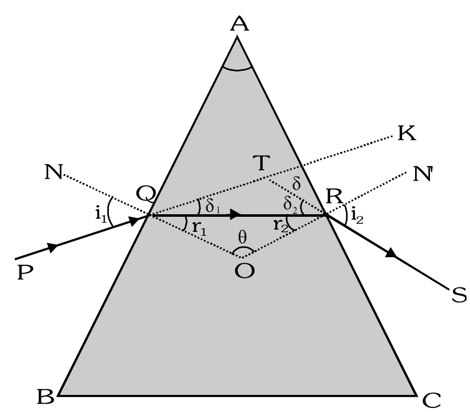


right angled isosceles prism



right angled prism

DEVIATION



PQ = incident ray

QR = Refracted ray

RS = emergent ray

A = Prism angle

i_1 = incident angle on face AB

i_2 = emergent angle on face AC

r_1 = refracted angle on face AB

r_2 = incident angle on face AC

Angle of deviation on face AB. $\delta_1 = i_1 - r_1$

Angle of deviation on face AC $\delta_2 = i_2 - r_2$

Total angle of deviation

$\delta = \delta_1 + \delta_2 \Rightarrow \delta = (i_1 - r_1) + (i_2 - r_2) = i_1 + i_2 - (r_1 + r_2)$ .....(i)

In ΔQOR $\quad r_1 + r_2 + \theta = 180$ ...(ii)

In AQOR $\quad A + \theta = 180$ ...(iii)

from (ii) and (iii) $\quad r_1 + r_2 = A$ ...(iv)

from (i) and (iv) $\quad$ Total angle of deviation $\delta = i_1 + i_2 - A$

from Snell's law at surface AB $\quad \mu_1 \sin i_1 = \mu_2 \sin r_1$

and at surface AC $\quad \mu_2 \sin r_2 = \mu_1 \sin i_2$